

Appendix J

Thevenin's Equation for Motor-Generators

Acknowledgment

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Thevenin's Equation

Thevenin's Equation provides a straightforward, easy-to-use and fairly accurate way to determine motor-generator torque for standard-type synchronous motor generators. It is normally written as:

$$T = \frac{q_1 V_{1a}^2 R_2}{S w_s \left[(R_{e1} + R_2/S)^2 + (X_{e1} + X_2)^2 \right]} \quad (\text{N}\cdot\text{m})$$

where

q_1	Number of Phases
V_{1a}	Equivalent Source Voltage (see definition below)
R_2	Rotor Resistance (Ω)
R_{e1}	Thevenin's Equivalent Stator Resistance (Ω)
S	Slip Ratio (see definition below)
X_2	Rotor Leakage Reactance (Ω)
X_{e1}	Thevenin's Equivalent Stator Leakage Reactance (Ω)
w_s	Synchronous Speed (see definition below)

In order to calculate R_{e1} and X_{e1} , we use the following formulae:

$$R_{e1} = \frac{R_1 X_{m'}^2}{R_1^2 + (X_1 + X_{m'})^2}$$
$$X_{e1} = \frac{X_{m'} \left[R_1^2 + X_1 (X_1 + X_{m'}) \right]}{R_1^2 + (X_1 + X_{m'})^2}$$

where

R_1	Stator Resistance (Ω)
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X_1 Stator Leakage Reactance (Ω)

$X_{m'}$ Magnetizing Reactance (Ω)

In order to calculate V_{1a} , the equivalent source voltage, we can use this relation:

$$V_{1A} = \frac{X_{m'} V_S}{\sqrt{R_1^2 + (X_1 + X_{m'})^2}}$$

where

$$V_S = \frac{V_{LL}}{\sqrt{3}}$$

and

V_{LL} Line-to-Line Voltage (volts-RMS)

We can calculate the synchronous speed, w_s , from the line frequency f , (in Hz) using the relation:

$$w_s = \frac{4 p f}{p}$$

where

p Number of Poles in motor-generator

Finally, the slip ratio, S , is defined as:

$$S = \frac{w_s - w}{w_s}$$

We can now do some rearranging in order to get an expression for torque in terms of the fixed characteristics and the two things we can measure, slip, S , and line voltage, V_{LL} . First, define two constants, K_1 and K_2 , as:

$$K_1 = (X_{e1} + X_2)^2$$

$$K_2 = \frac{X_{m'}^2}{R_1^2 + (X_1 + X_{m'})^2}$$

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We can then write

$$T = \frac{q_1 R_2 K_2 V_{LL}^2 S}{3w_s \left[\left(R_{e_1}^2 + K_1 \right) S^2 + 2R_{e_1} R_2 S + R_2^2 \right]}$$

This gives us a fairly useful expression for motor-generator torque in terms of slip, speed and four coefficients:

$$T = \frac{A_0 V_{LL}^2 S}{C_0 + C_1 S + C_2 S^2}$$

where

$$\begin{aligned} A_0 &= \frac{q_1 R_2 K_2}{3w_s} \\ C_0 &= R_2^2 \\ C_1 &= 2R_{e_1} R_2 \\ C_2 &= \left(R_{e_1}^2 + K_1 \right) \end{aligned}$$

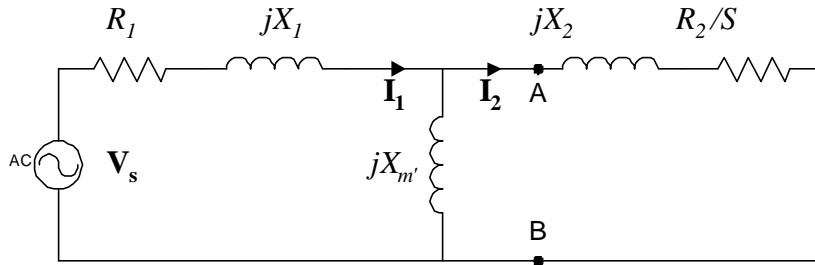
Remembering again that

$$\begin{aligned} K_1 &= (X_{e_1} + X_2)^2 \\ K_2 &= \frac{X_{m'}^2}{R_1^2 + (X_1 + X_{m'})^2} \end{aligned}$$

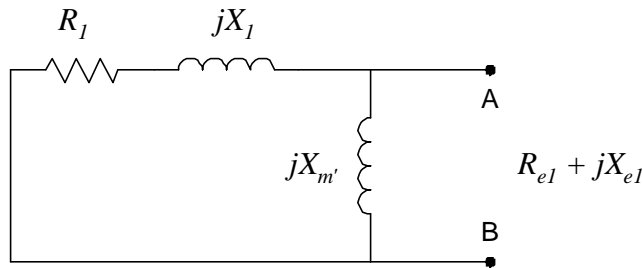
Appendix J - Thevenin's Equation

The following four circuit diagrams show the steps used to develop Thevenin's Equivalent Circuit:

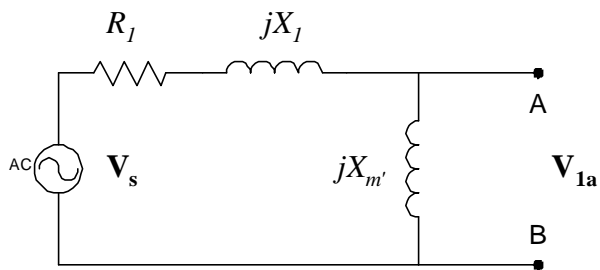
Complete Circuit



Thevenin Impedance



Open Circuit Voltage



Thevenin's Equivalent Circuit

